Available online at www.sciencedirect.com
ScienceDirect
Journal of Taibah University for Science xxx (2016) xxx–xxx
Journal of Taibah University for Science
www.elsevier.com/locate/jtusci

Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability

Aly R. Seadawy [a,b,*], Dianchen Lu [c], Chen Yue [c]

[a] *Mathematics Department, Faculty of Science, Taibah University, Al-Ula, Saudi Arabia*
[b] *Mathematics Department, Faculty of Science, Beni-Suef University, Egypt*
[c] *Department of Mathematics, Faculty of Science, Jiangsu University, China*

Received 17 January 2016; received in revised form 9 April 2016; accepted 15 June 2016

Abstract

In the present study, by implementing the direct algebraic method, we present the traveling wave solutions for some different kinds of the Korteweg–de Vries (KdV) equations. The exact solutions of the Kawahara, fifth order KdV and generalized fifth order KdV equations are obtained. Solutions for the Kawahara, fifth order KdV and generalized fifth order KdV equations are obtained precisely and efficiency of the method can be demonstrated. The stability of these solutions and the movement role of the waves by making the graphs of the exact solutions are analyzed. All solutions are exact and stable, and have applications in physics.

Keywords: Traveling wave solutions; Extended direct algebraic method; Kawahara; Fifth order KdV; Generalized KdV equations

1. Introduction

Nonlinear wave phenomena exist in many fields, such as fluid mechanics, plasma physics, biology, hydrodynamics, solid state physics and optical fibers [1–4]. In order to better understand these nonlinear phenomena, it is important to seek their exact solutions. They can help to analyze the stability of these solutions and the movement role of the wave by making the graphs of the exact solutions [5–9]. The KdV equation plays an important role in describing motions of long waves in shallow water under gravity, one-dimensional nonlinear lattice, fluid mechanics, quantum mechanics, plasma physics and nonlinear optics [9–12].

There are many classical methods proposed to solve the KdV equations, including direct integration, direct algebraic approach, Lyapunov approach, Hirota's dependent variable transformation, the inverse scattering transform, and the

* Corresponding author.
E-mail address: aly742001@yahoo.com (A.R. Seadawy).
Peer review under responsibility of Taibah University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jtusci.2016.06.002

Bäcklund transformation. Analytical exact solutions of a nonlinear evolution equation in mathematical physics; namely the time-fractional fifth-order Sawada–Kotera equation by the tanh–sech method via fractional complex transform were constructed [5]. Wazwaz considered the abundant solitons solutions, compactons and solitary patterns solutions, some new solitons and periodic solutions of the fifth-order KdV equation [13,14]. Sierra and Salas used a generalization of the well-known tanh–coth method to obtain new periodic and soliton solutions for several forms of the fifth-order KdV equation [15]. Using the Lie group analysis method, the invariance properties of the time fractional generalized fifth-order KdV equation were studied [16].

An extended simplest equation method was proposed to seek exact travelling wave solutions of nonlinear evolution equations. As applications, many new exact travelling wave solutions for several forms of the fifth-order KdV equation were obtained. The forms include the Lax, Sawada–Kotera, Sawada–Kotera–Parker–Dye, Caudrey–Dodd–Gibbon, Kaup–Kupershmidt, Kaup–Kupershmidt–Parker–Dye, and the Ito forms [17]. The Exp-function method was applied to obtain new generalized solitary solutions and periodic solutions of the fifth-order KdV equation [18–22]. The 1-soliton solution of three variants of the generalized KdV equation with generalized evolution were obtained [23,24]. Soliton solutions to KdV equation with spatio-temporal dispersion were given [25]. Additional conservation laws for Rosenau–KdV–RLW equation with power law nonlinearity by Lie symmetry was obtained [26]. Exact and explicit solutions to some nonlinear evolution equations by utilizing the (G'/G)-expansion and extended direct algebraic methods were given [27–31].

This paper is organized as follows: an introduction is given in Section 1. In Section 2, an analysis of the extended direct algebraic method is formulated. In Sections 3 and 4, the exact solutions of the Kawahara, the fifth order KdV and generalized fifth order KdV equations are obtained. Finally the paper end with a conclusion is given in Section 5.

2. An analysis of the extended direct algebraic method

The following is a given nonlinear partial differential equations (KdV equations) with two variables x and t [32] as

$$F(u, \quad u_t, \quad u_x, \quad u_{xx}, \quad u_{xxx}, \quad u_{xxxxx}) = 0, \tag{1}$$

where F is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformations.

Step 1: Assume that Eq. (1) has the following formal solution as:

$$u(x, t) = u(\xi) = \sum_{i=0}^{m} a_i \varphi^i(\xi), \tag{2}$$

where

$$\varphi\prime = \sqrt{\alpha\varphi^2 + \beta\varphi^4} \quad \text{and} \quad \xi = kx + \omega t, \tag{3}$$

where α, β, are arbitrary constants and k and ω are the wave length and frequency.

Step 2: Balancing the highest order derivative term and the highest order nonlinear term of Eq. (1), and the coefficients of series α, β, a_0, a_1, a_m, k, ω are parameters can be determined.

Step 3: Substituting from Eqs. (2) and (3) into Eq. (1) and collecting coefficients of $\varphi^i\ \varphi^{(i)}$, then setting coefficients equal zero, we will obtain a set of algebraic equations. By solving the system, the parameters α, β, a_0, a_1, a_m, k, ω can be determined.

Step 4: By substituting the parameters α, β, a_0, a_1, a_m, k, ω and $\varphi(\xi)$ obtained in step 3 into Eq. (2), the solutions of Eq. (1) can be derived.

3. Stability analysis

Hamiltonian system is a mathematical formalism to describe the evolution equations of a physical system. By using the form of a Hamiltonian system for which the momentum is given as

$$M = \frac{1}{2} \int_{-\infty}^{\infty} u^2 d\xi, \tag{4}$$

where M is the momentum, u is the travelling wave solutions in Eq. (2). The sufficient condition for soliton stability is

$$\frac{\partial M}{\partial \omega} > 0, \tag{5}$$

where ω is the frequency. By using Eqs. (4) and (5), we discuss the criteria for stability of solutions for the generalized fifth order KdV equations in given intervals.

4. The applications of the method

4.1. The Kawahara equation

The Kawahara equation is a generic approximation equation describing the evolution of long waves in situations in which the coefficient in front of the third derivative term in the KdV equation becomes very small. One situation in which this occurs is for waves on the surface of an inviscid, incompressible fluid undergoing irrotational motion, subject to surface tension on the free surface. The Kawahara equation arises in the special, but interesting, parameter regime when the Bond number, a dimensionless parameter proportional to the surface tension, is nearly equal to 1/3. Consider the Kawahara equation, which occurs in the theory of shallow water waves with surface tension [33] as

$$u_t + uu_x - u_{xxx} + u_{xxxxx} = 0. \tag{6}$$

Consider the traveling wave solutions (2) and (3), then Eq. (6) becomes

$$\omega u' + kuu' - k^3 u^{(3)} + k^5 u^{(5)} = 0. \tag{7}$$

Balancing the nonlinear term uu' and the highest order derivative $u^{(5)}$ in Eq. (13) gives $m = 4$. Suppose the solution of Eq. (7) in the form

$$u(\xi) = a_0 + a_1\varphi + a_2\varphi^2 + a_3\varphi^3 + a_4\varphi^4. \tag{8}$$

By substituting (8) into Eq. (7) yields a set of algebraic equations for a_0, a_1, a_2, a_3, a_4, α, β, k, ω. The solution of the system of algebraic equations, can be found as

$$a_0 = -\frac{36k + 169\omega}{169k}, \quad a_1 = 0, \quad a_2 = 0, \quad a_3 = 0, \quad a_4 = 1680k^4\beta^2, \quad \alpha = \frac{1}{52k^2}. \tag{9}$$

Substituting from Eqs. (9) into (8), the following solutions of Eq. (6) can be obtained as

$$u_1(x, t) = -\frac{36k + 169\omega}{169k} + \frac{1680k^3\beta\sqrt{\beta}}{2\sqrt{13}} \quad sech^4\left(\frac{1}{2k\sqrt{13}}(kx + \omega t)\right), \tag{10}$$

$$u_2(x, t) = -\frac{36k + 169\omega}{169k} + \frac{159\beta^2 \quad \exp((2/k\sqrt{13})(kx + \omega t))}{\left(1 - 4\beta \quad \exp((1/2k\sqrt{13})(kx + \omega t))\right)^4}, \tag{11}$$

$$u_3(x, t) = -\frac{36k + 169\omega}{169k} + \frac{159\beta^2 \quad \exp((2/k\sqrt{13})(kx + \omega t))}{\left(\exp((1/2k\sqrt{13})(kx + \omega t)) - 4\beta\right)^4}. \tag{12}$$

The traveling wave solutions (10)–(12) are shown in Fig. 1a–d, with $k = 0.5$, $\beta = 4$ and $\omega = -0.8$ in the interval $[-10, 10]$ and $[0, 5]$. According to the conditions of stability (4) and (5), the travelling wave solutions (10)–(12) are stable in the interval $[-10, 10]$ and $[0, 5]$.

4.2. The fifth order KdV equation (Case I)

Consider the first case of the fifth order KdV equation as [32,34]

$$u_t + uu_x - uu_{xxx} + u_{xxxxx} = 0. \tag{13}$$

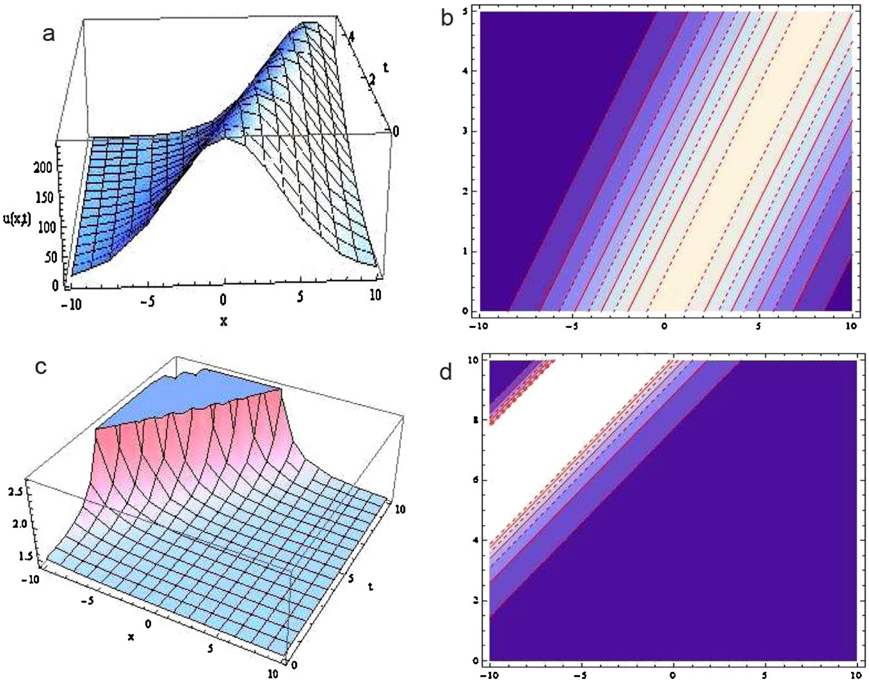

Fig. 1. Travelling waves solutions (10)–(12) with various different shapes are plotted: bright solitary waves in (a), contour plot in (b). Travelling waves solutions (10)–(12) with various different shapes are plotted: solitary waves in (c), contour plot in (d).

Consider the traveling wave solutions (2) and (3), then Eq. (13) becomes

$$\omega u' + kuu' - k^3 uu^{(3)} + k^5 u^{(5)} = 0. \tag{14}$$

Balancing the nonlinear term $uu^{(3)}$ and the highest order derivative $u^{(5)}$ in Eq. (14) gives $m = 2$. Suppose the solution of Eq. (14) in the form

$$u(\xi) = a_0 + a_1\varphi + a_2\varphi^2. \tag{15}$$

By substituting from (15) into Eq. (14) yields a set of algebraic equations for a_0, a_1, a_2, α, β, k, ω. The solution of this system of equations, can be found as

$$\begin{aligned} &\omega = \frac{1}{2}(-5k + 48k^5\alpha^2), \quad a_0 = \frac{5}{2}(1 + 4k^2\alpha), \quad a_1 = 0, \quad a_2 = 30k^2\beta, \\ &\omega = -\frac{35k}{54}, \quad a_0 = \frac{95}{18}, \quad a_1 = 0, \quad a_2 = 30k^2\beta, \quad \alpha = \frac{5}{18k^2}. \end{aligned} \tag{16}$$

By substituting from (16) into Eq. (15), we have obtained the following solutions of Eq. (13) as

$$u_1(x,t) = \frac{5}{2}(1 + 4k^2\alpha) + 30k^2\sqrt{\alpha\beta}\ \ sech^2\left(\sqrt{\alpha}\left(kx + \left(\frac{1}{2}(-5k + 48k^5\alpha^2)\right)t\right)\right), \tag{17}$$

$$u_2(x,t) = \frac{5}{2}(1 + 4k^2\alpha) + \frac{480\alpha\beta k^2 \quad \exp(2\sqrt{\alpha}(kx + \left((1/2)(-5k + 48k^5\alpha^2)\right)t))}{\left(1 - 4\beta \quad \exp(\sqrt{\alpha}(kx + \left((1/2)(-5k + 48k^5\alpha^2)\right)t))\right)^2}, \tag{18}$$

$$u_3(x,t) = \frac{5}{2}(1 + 4k^2\alpha) + \frac{480\alpha\beta k^2 \quad \exp\left(2\sqrt{\alpha}\left(kx + \left(1/2(-5k + 48k^5\alpha^2)\right)t\right)\right)}{\left(\exp(\sqrt{\alpha}(kx + \left(1/2(-5k + 48k^5\alpha^2)\right)t)) - 4\beta\right)^2}, \tag{19}$$

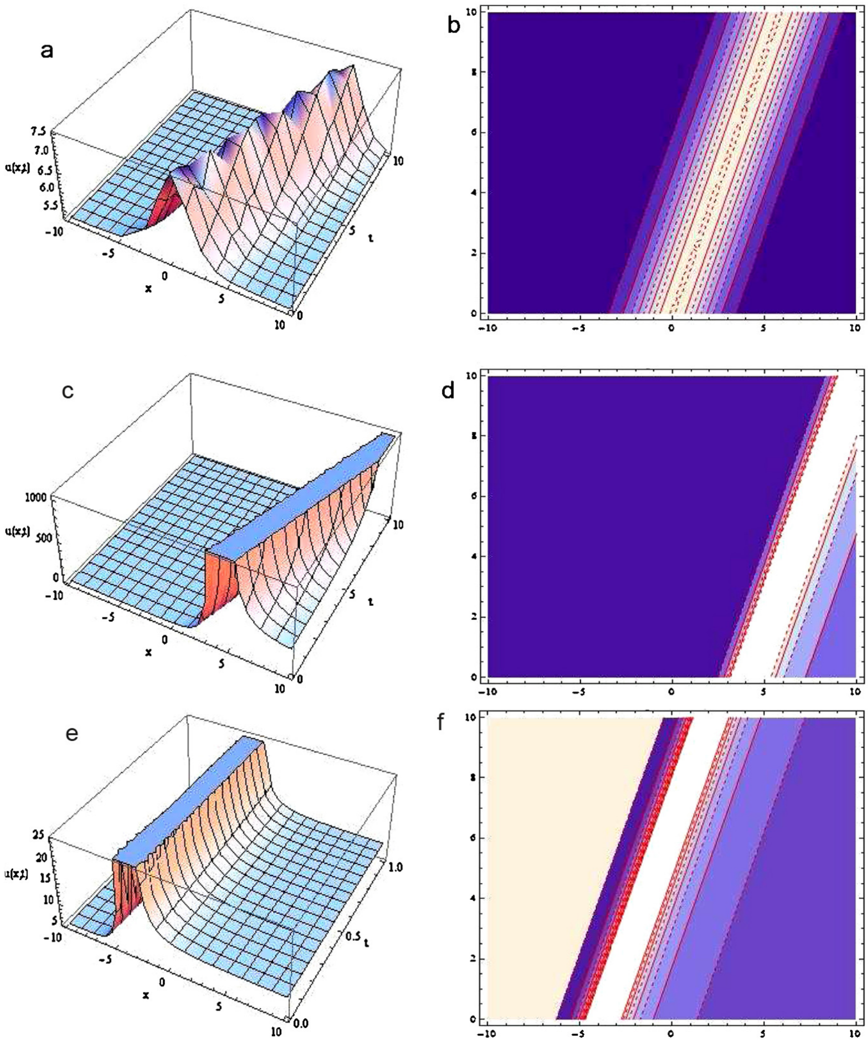

Fig. 2. Travelling waves solutions (17) with various different shapes are plotted: solitary waves in (a), contour plot in (b). Travelling waves solutions (18) and (19) with various different shapes are plotted: solitary waves in (c), contour plot in (d). Travelling waves solutions (20)–(22) with various different shapes are plotted: solitary waves in (e), contour plot in (f).

$$u_4(x,t) = \frac{95}{18} + 10k\sqrt{\frac{5\beta}{2}} \quad sech^2\left(\frac{1}{3}\sqrt{\frac{5}{2}}\left(x - \frac{35}{54}t\right)\right), \tag{20}$$

$$u_5(x,t) = \frac{95}{18} + \frac{400\beta \quad \exp\left((1/3)\sqrt{10}\,(x-(35/54)t)\right)}{3\left(1-4\beta \quad \exp\left((1/3)\sqrt{(5/2)}\,(x-(35/54)t)\right)\right)^2}, \tag{21}$$

$$u_6(x,t) = \frac{95}{18} + \frac{400\beta \quad \exp\left((1/3)\sqrt{10}\,(x-(35/54)t)\right)}{3\left(\exp\left((1/3)\sqrt{(5/2)}\,(x-(35/54)t)\right)-4\beta\right)^2}. \tag{22}$$

Fig. 2a–f represents the traveling wave solutions (17)–(22), with $k = 0.9$ and $\beta = 2$ in the interval $[-10, 10]$ and $[0, 10]$. According to the conditions of stability (4) and (5), the travelling wave solutions (17)–(22) are stable in the interval $[-10, 10]$ and $[0, 10]$.

Please cite this article in press as: A.R. Seadawy, et al. Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability, J. Taibah Univ. Sci. (2016), http://dx.doi.org/10.1016/j.jtusci.2016.06.002

4.3. The fifth order KdV equation (Case II)

Consider the second case of the fifth order KdV equation as [32,34]

$$u_t+(1+u)u_x+(1+u)u_{xxx}+u_{xxxxx}=0. \tag{23}$$

Consider the traveling wave solutions (2) and (3), then Eq. (23) becomes

$$(\omega+k)u'+kuu'+k^3u^{(3)}+k^3uu^{(3)}+k^5u^{(5)}=0. \tag{24}$$

Balancing the nonlinear term $uu^{(3)}$ and the highest order derivative $u^{(5)}$ in Eq. (24) gives $m=2$. Suppose the solution of Eq. (24) in the form

$$u(\xi)=a_0+a_1\varphi+a_2\varphi^2. \tag{25}$$

Substituting from Eq. (25) into Eq. (24) yields a set of algebraic equations for a_0, a_1, a_2, α, β, k, ω. The solution of the system of algebraic equations, can be obtained as

$$\omega=\frac{1}{2}(-5k+48k^5\alpha^2),\quad a_0=\frac{1}{2}(3-20k^2\alpha),\quad a_1=0,\quad a_2=-30k^2\beta,\quad \omega=\frac{7k}{2},$$

$$a_0=-\frac{7}{2},\quad a_1=0,\quad a_2=-30k^2\beta,\quad \alpha=\frac{1}{2k^2}. \tag{26}$$

By substituting from Eqs. (26) into (25), the following solutions of Eq. (23) can be obtained as

$$u_1(x,t)=(\frac{1}{2})(3-20k^2\alpha)-30k^2\sqrt{\alpha\beta}\ \ sech^2\left(\sqrt{\alpha}\left(kx+\left(\frac{1}{2}\right)(-5k+48k^5\alpha^2)t\right)\right), \tag{27}$$

$$u_2(x,t)=\left(\frac{1}{2}\right)(3-20k^2\alpha)-\frac{480\alpha\beta k^2\ \ \exp\left(2\sqrt{\alpha}\left(kx+(1/2)(-5k+48k^5\alpha^2)t\right)\right)}{\left(1-4\beta\ \ \exp\left(\sqrt{\alpha}\left(kx+(1/2)(-5k+48k^5\alpha^2)t\right)\right)\right)^2}, \tag{28}$$

$$u_3(x,t)=(\frac{1}{2})(3-20k^2\alpha)-\frac{480\alpha\beta k^2\ \ \exp\left(2\sqrt{\alpha}\left(kx+(1/2)(-5k+48k^5\alpha^2)t\right)\right)}{\left(\exp\left(\sqrt{\alpha}\left(kx+(1/2)(-5k+48k^5\alpha^2)t\right)\right)-4\beta\right)^2}, \tag{29}$$

$$u_4(x,t)=-\left(\frac{7}{2}\right)-30k\sqrt{\frac{\beta}{2}}\ \ sech^2\left(\frac{1}{\sqrt{2}}(x+(7/2)t)\right), \tag{30}$$

$$u_5(x,t)=-\left(\frac{7}{2}\right)-\frac{240\beta\ \ \exp\left(\sqrt{2}\,(x+(7/2)t)\right)}{\left(1-4\beta\ \ \exp\left((1/\sqrt{2})\,(x+(7/2)t)\right)\right)^2}, \tag{31}$$

$$u_6(x,t)=-\left(\frac{7}{2}\right)-\frac{240\beta\ \ \exp\left(\sqrt{2}\,(x+(7/2)t)\right)}{\left(\exp\left((1/\sqrt{2})\,(x+(7/2)t)\right)-4\beta\right)^2}. \tag{32}$$

The traveling wave solutions (27)–(32) are shown in Fig. 3a–f, with $k=0.5$ and $\beta=2$ in the interval [−10, 10] and [0, 5]; [−8, 8] and [0, 8]. According to the conditions of stability (4) and (5), the travelling wave solutions (27)–(32) are stable in the interval [−10, 10] and [0, 5]; [−8, 8] and [0, 8].

4.4. The generalized fifth order KdV equation

Consider the generalized fifth order KdV equation as [35,36]

$$u_t+45u^2u_x-\lambda u_xu_{xx}-15uu_{xxx}+u_{xxxxx}=0. \tag{33}$$

Now, two important unidirectional nonlinear evolution equations that have been studied extensively over the last two decades are the Sawada–Kotera (SK) (or Caudrey–Dodd–Gibbon) equation, if $\lambda=15$ [37,38]

$$u_t+45u^2u_x-15u_xu_{xx}-15uu_{xxx}+u_{xxxxx}=0, \tag{34}$$

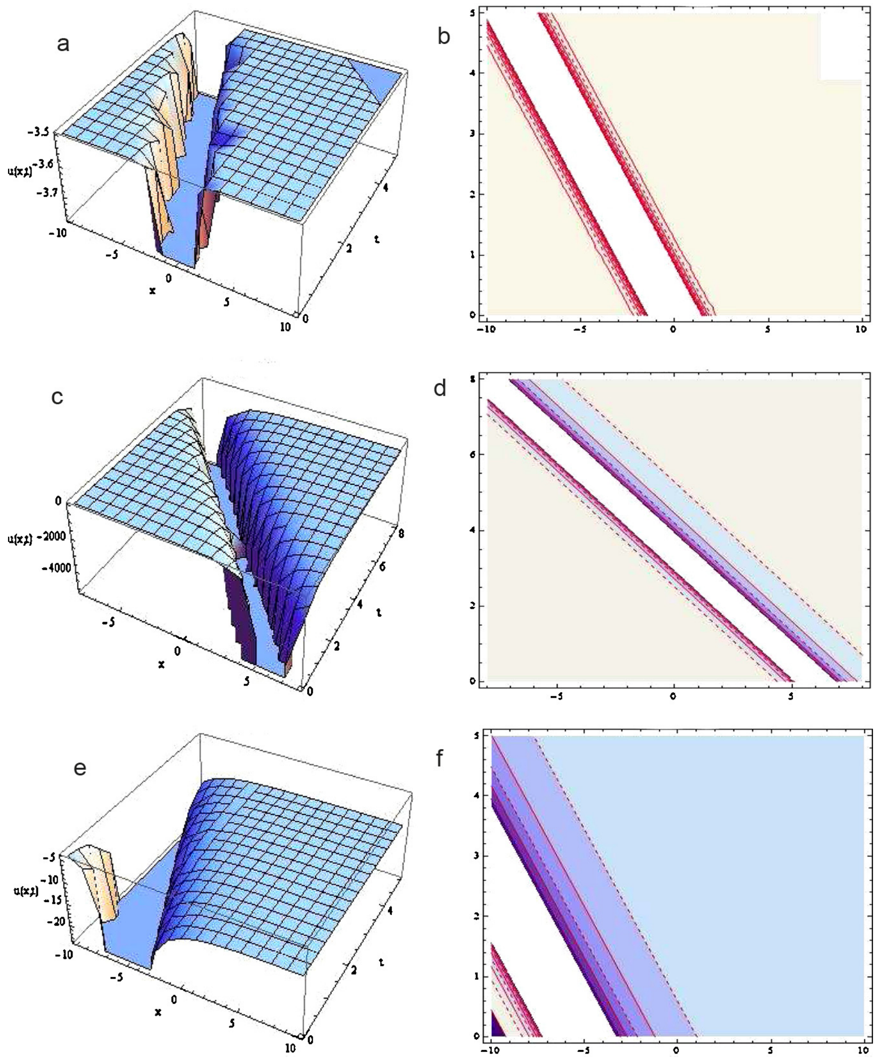

Fig. 3. Travelling waves solutions (30) with various different shapes are plotted: dark solitary waves in (a), contour plot in (b). Travelling waves solutions (31) with various different shapes are plotted: dark solitary waves in (c), contour plot in (d). Travelling waves solutions (32) with various different shapes are plotted: solitary waves in (e), contour plot in (f).

and the Kaup–Kupershmidt (KK) equation, if $\lambda = 75/2$ [39,40]

$$u_t + 45u^2u_x - \frac{75}{2}u_xu_{xx} - 15uu_{xxx} + u_{xxxxx} = 0. \tag{35}$$

Consider the traveling wave solutions (2) and (3), then Eq. (33) becomes

$$\omega u' + 45ku^2u' - \lambda k^3u'u'' - 15k^3uu^{(3)} + k^5u^{(5)} = 0. \tag{36}$$

Balancing the nonlinear term $uu^{(3)}$ and the highest order derivative $u^{(5)}$ gives $m = 2$. Suppose the solution of Eq. (36) is of the form

$$u(\xi) = a_0 + a_1\varphi + a_2\varphi^2. \tag{37}$$

By substituting from Eq. (37) into Eq. (36) yields a set of algebraic equations for a_0, a_1, a_2, α, β, k, ω. We have two cases of the solutions of these algebraic equations as

Case I: If $\lambda = 15$, the solution of the system of equations, can be obtained as

$$a_0 = \frac{4k^2\alpha}{3}, \quad a_1 = 0, \quad a_2 = 4k^2\beta, \quad \omega = -16k^5\alpha^2,$$
$$a_0 = \frac{13k^2\alpha}{15}, \quad a_1 = 0, \quad a_2 = 2k^2\beta, \quad \omega = \frac{11k^5\alpha^2}{5}, \tag{38}$$

Substituting from Eq. (38) into (37), the following solution of the Sawada–Kotera Eq. (34) can be obtained as

$$u_1(x,t) = \frac{4k^2\alpha}{3} + 4k^2\sqrt{\alpha\beta} \quad sech^2\left(\sqrt{\alpha}(kx - 16k^5\alpha^2 t)\right), \tag{39}$$

$$u_2(x,t) = \frac{4k^2\alpha}{3} + \frac{64\alpha\beta k^2 \quad \exp(2\sqrt{\alpha}(kx - 16k^5\alpha^2 t))}{\left(1 - 4\beta \quad \exp(\sqrt{\alpha}(kx - 16k^5\alpha^2 t))\right)^2}, \tag{40}$$

$$u_3(x,t) = \frac{4k^2\alpha}{3} + \frac{64\alpha\beta k^2 \quad \exp(2\sqrt{\alpha}(kx - 16k^5\alpha^2 t))}{\left(\exp(\sqrt{\alpha}(kx - 16k^5\alpha^2 t)) - 4\beta\right)^2}, \tag{41}$$

$$u_4(x,t) = \frac{13k^2\alpha}{15} + 2k^2\sqrt{\alpha\beta} \quad sech^2\left(\sqrt{\alpha}\left(kx + \frac{11k^5\alpha^2}{5}t\right)\right), \tag{42}$$

$$u_5(x,t) = \frac{13k^2\alpha}{15} + \frac{32\alpha\beta k^2 \quad \exp\left(2\sqrt{\alpha}\left(kx + (11k^5\alpha^2/5)t\right)\right)}{\left(1 - 4\beta \quad \exp(\sqrt{\alpha}(kx + (11k^5\alpha^2/5)t))\right)^2}, \tag{43}$$

$$u_6(x,t) = \frac{13k^2\alpha}{15} + \frac{32\alpha\beta k^2 \quad \exp\left(2\sqrt{\alpha}\left(kx + (11k^5\alpha^2/5)t\right)\right)}{\left(\exp\left(\sqrt{\alpha}\left(kx + (11k^5\alpha^2/5)t\right)\right) - 4\beta\right)^2}. \tag{44}$$

Case II: If $\lambda = 75/2$, the solution of the system of equations, can be found as

$$a_0 = \frac{8k^2\alpha}{3}, \quad a_1 = 0, \quad a_2 = 8k^2\beta, \quad \omega = -176k^5\alpha^2,$$
$$a_0 = \frac{k^2\alpha}{3}, \quad a_1 = 0, \quad a_2 = k^2\beta, \quad \omega = -k^5\alpha^2. \tag{45}$$

Substituting from Eq. (45) into (37), the following solution of the Kaup–Kupershmidt Eq. (35) can be obtained as

$$u_1(x,t) = \frac{8k^2\alpha}{3} + 8k^2\sqrt{\alpha\beta} \quad sech^2\left(\sqrt{\alpha}(kx - 176k^5\alpha^2 t)\right), \tag{46}$$

$$u_2(x,t) = \frac{8k^2\alpha}{3} + \frac{128\alpha\beta k^2 \quad \exp\left(2\sqrt{\alpha}\left(kx - 176k^5\alpha^2 t\right)\right)}{\left(\exp(\sqrt{\alpha}(kx - 176k^5\alpha^2 t)) - 4\beta\right)^2}, \tag{47}$$

$$u_3(x,t) = \frac{8k^2\alpha}{3} + \frac{128\alpha\beta k^2 \quad \exp(2\sqrt{\alpha}(kx - 176k^5\alpha^2 t))}{\left(1 - 4\beta \quad \exp\left(\sqrt{\alpha}\left(kx - 176k^5\alpha^2 t\right)\right)\right)^2}, \tag{48}$$

$$u_4(x,t) = \frac{k^2\alpha}{3} + k^2\sqrt{\alpha\beta} \quad sech^2\left(\sqrt{\alpha}(kx - k^5\alpha^2 t)\right), \tag{49}$$

$$u_5(x,t) = \frac{k^2\alpha}{3} + \frac{16\alpha\beta k^2 \quad \exp\left(2\sqrt{\alpha}\left(kx - k^5\alpha^2 t\right)\right)}{\left(1 - 4\beta \quad \exp\left(\sqrt{\alpha}\left(kx - k^5\alpha^2 t\right)\right)\right)^2}, \tag{50}$$

$$u_6(x,t) = \frac{k^2\alpha}{3} + \frac{16\alpha\beta k^2 \quad \exp\left(2\sqrt{\alpha}\left(kx - k^5\alpha^2 t\right)\right)}{\left(\exp(\sqrt{\alpha}(kx - k^5\alpha^2 t)) - 4\beta\right)^2}. \tag{51}$$

Fig. 4a–d represent the traveling wave solutions $u(x, t)$ of Eqs. (39)–(44), with $k = 1$, $\alpha = 4$ and $\beta = 1$ in the interval $[-1, 1]$ and $[0, 0.01]$; and Eqs. (46)–(51), with $k = -2$, $\alpha = 4$ and $\beta = 1$ in the interval $[-1, 1]$ and $[0, 0.01]$. According

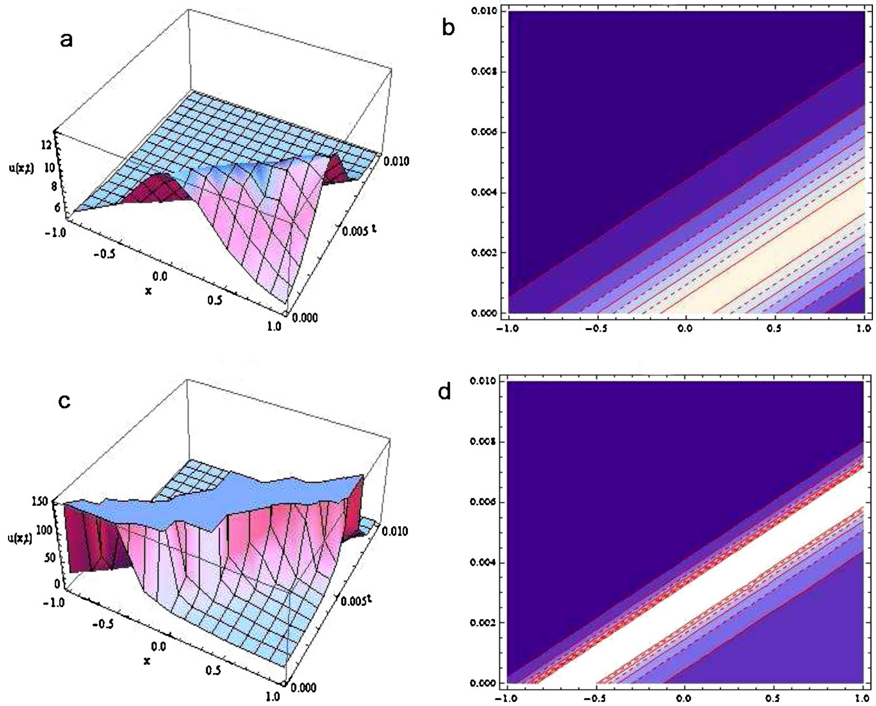

Fig. 4. Travelling waves solutions (39) with various different shapes are plotted: bright solitary waves in (a), contour plot in (b). Travelling waves solutions (50) with various different shapes are plotted: solitary waves in (c), contour plot in (d).

to the conditions of stability (4) and (5), the travelling wave solutions (39)–(44) and (46)–(51) are stable in the interval $[-1, 1]$ and $[0, 0.01]$.

5. Conclusion

An analytic study was conducted on coupled partial differential equations. We formally derived one soliton solutions for each Kawahara equation, fifth order KdV equation and generalized fifth order KdV equation. However, using another distinct approach, we derived traveling wave solutions for each equations. The structures of the obtained solutions are distinct and all these solutions are completely new and stable.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors drafted the manuscript, and they read and approved the final manuscript.

Acknowledgements

This manuscript was supported by the Deanship of Scientific Research, Taibah University, KSA; and Department of Mathematics, Faculty of Science, Jiangsu University, China.

References

[1] A.R. Seadawy, Stability analysis for Zakharov–Kuznetsov equation of weakly nonlinear ion-acoustic waves in a plasma, Comput. Math. Appl. 67 (2014) 172–180.

Please cite this article in press as: A.R. Seadawy, et al. Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability, J. Taibah Univ. Sci. (2016), http://dx.doi.org/10.1016/j.jtusci.2016.06.002

[2] A.R. Seadawy, Stability analysis for two-dimensional ion-acoustic waves in quantum plasmas, Phys. Plasmas 21 (2014) 052107.
[3] A.H. Khater, D.K. Callebaut, W. Malfliet, A.R. Seadawy, Nonlinear dispersive Rayleigh–Taylor instabilities in magnetohydro-dynamic flows, Phys. Scr. 64 (2001) 533–547.
[4] A.H. Khater, D.K. Callebaut, A.R. Seadawy, Nonlinear dispersive Kelvin–Helmholtz instabilities in magnetohydrodynamic flows, Phys. Scr. 67 (2003) 340–349.
[5] S. Saha Ray, S. Sahoo, A novel analytical method with fractional complex transform for new exact solutions of time-fractional fifth-order Sawada–Kotera equation, Rep. Math. Phys. 75 (2015) 63–72.
[6] M.A. Helal, A.R. Seadawy, Benjamin–Feir-instability in nonlinear dispersive waves, Comput. Math. Appl. 64 (2012) 3557–3568.
[7] A.R. Seadawy, The solutions of the boussinesq and generalized fifth-order KdV equations by using the direct algebraic method, Appl. Math. Sci. 6 (2012) 4081–4090.
[8] Turgut Ozis, Ismail Aslan, Application of the G'/G-expansion method to Kawahara type equations using symbolic computation, Appl. Math. Comput. 216 (2010) 2360–2365.
[9] A.R. Seadawy, New exact solutions for the KdV equation with higher order nonlinearity by using the variational method, Comput. Math. Appl. 62 (2011) 3741–3755.
[10] Ismail Aslan, Turgut Ozis, Analytic study on two nonlinear evolution equations by using the (G'/G)-expansion method, Appl. Math. Comput. 209 (2009) 425–429.
[11] Houria Triki, Anjan Biswas, Soliton solutions for a generalized fifth-order KdV equation with time-dependent coefficients, Waves Random Complex Media 21 (2011) 151–160.
[12] M.A. Helal, Soliton solution of some nonlinear partial differential equations and its applications in fluid mechanics, Chaos Solitons Fractals 13 (2002) 1917–1929.
[13] A.M. Wazwaz, Compactons and solitary patterns solutions to fifth-order KdV-like equations, Physica A 371 (2006) 273–279.
[14] A.M. Wazwaz, Abundant solitons solutions for several forms of the fifth order KdV equation by using the tanh method, Appl. Math. Comput. 182 (2006) 283–300.
[15] C.A.G. Sierra, A.H. Salas, The generalized tanh–coth method to special types of the fifth-order KdV equation, Appl. Math. Comput. 203 (2008) 873–880.
[16] G.-W. Wang, X.-Q. Liu, Y.-Y. Zhang, Lie symmetry analysis to the time fractional generalized fifth-order KdV equation, Commun. Nonlinear Sci. Numer. Simul. 18 (2013) 2321–2326.
[17] S. Bilige, T. Chaolu, An extended simplest equation method and its application to several forms of the fifth-order KdV equation, Appl. Math. Comput. 216 (2010) 3146–3153.
[18] Muhammad Aslam Noor, Syed Tauseef Mohyud-Din, Asif Waheed, Eisa A. Al-Said, Exp-function method for traveling wave solutions of nonlinear evolution equations, Appl. Math. Comput. 216 (2010) 477–483.
[19] A.R. Seadawy, K. El-Rashidy, Classification of travelling wave solutions for coupled Burgers equations, combined KdV-modified KdV and Schrodinger–KdV equations, Abstr. Appl. Anal. 369294 (2015) 1–7.
[20] S.T. Mohyud-Din, M.A. Noor, K.I. Noor, Traveling wave solutions of seventh-order generalized KdV equations using He's polynomials, Int. J. Nonlinear Sci. Numer. Simul. 10 (2) (2009) 223–229.
[21] Muhammad Aslam Noor, Syed Tauseef Mohyud-Din, Asif Waheed, Exp-function method for generalized traveling solutions of master partial differential equations, Acta Appl. Math. 104 (2) (2008) 131–137.
[22] Syed Tauseef Mohyud-Din, Muhammad Aslam Noor, Asif Waheed, Exp-function method for generalized traveling solutions of Calogero–Degasperis–Fokas equation, Z. Naturforschung A[J. Phys. Sci.] 65a (2010) 78–84.
[23] Muhmmad S. Ismail, Anjan Biswas, 1-Soliton solution of the generalized KdV equation with generalized evolution, Appl. Math. Comput. 216 (2010) 1673–1679.
[24] Mohammad Mirzazadeh, M. Eslami, Anjan Biswas, 1-Soliton solution to KdV6 equation, Nonlinear Dyn. 80 (2015) 387–396.
[25] Houria Triki, A.K. Turgut, Seithuti Moshokoa, Anjan Biswas, Soliton solutions to KdV equation with spatio-temporal dispersion, Ocean Eng. 114 (2016) 192–203.
[26] Polina Razborova, Abdul H. Kara, Anjan Biswas, Additional conservation laws for Rosenau–KdV–RLW equation with power law nonlinearity by lie symmetry, Nonlinear Dyn. 79 (2015) 743–748.
[27] Ismail Aslan, On the application of the Exp-function method to the KP equation for N-soliton solutions, Appl. Math. Comput. 219 (6) (2012) 2825–2828.
[28] Ismail Aslan, Exact and explicit solutions to some nonlinear evolution equations by utilizing the (G'/G)-expansion method, Appl. Math. Comput. 215 (2010) 857–863.
[29] A.R. Seadawy, Stability analysis of traveling wave solutions for generalized coupled nonlinear KdV equations, Appl. Math. Inf. Sci. 10 (1) (2016) 209–214.
[30] A.R. Seadawy, Stability analysis solutions for nonlinear three-dimensional modified Korteweg–de Vries–Zakharov–Kuznetsov equation in a magnetized electron-positron plasma, Physica A 455 (2016) 44–51.
[31] A.R. Seadawy, K. El-Rashidy, Traveling wave solutions for some coupled nonlinear evolution equations by using the direct algebraic method, Math. Comput. Model. 57 (2013) 1371.
[32] Y. Ugurlu, I.E. Inan, Exact and numerical solutions of the fifth order KdV equations and couple KdV system by using to direct algebraic method, Turk. J. Sci. Technol. 4 (2009) 45–56.
[33] J.K. Hunter, J. Scheurle, Existence of perturbed solitary wave solutions to a model equation for water waves, Physica D 32 (1988) 253–268.
[34] D. Kaya, An explicit and numerical solutions of some fifth-order KdV equation by decomposition method, Appl. Math. Comput. 144 (2003) 353–363.
[35] G.B. Whitham, Linear and Nonlinear Waves, John Wiley, New York, 1974.

Please cite this article in press as: A.R. Seadawy, et al. Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability, J. Taibah Univ. Sci. (2016), http://dx.doi.org/10.1016/j.jtusci.2016.06.002

[36] A.R. Seadawy, Fractional solitary wave solutions of the nonlinear higher-order extended KdV equation in a stratified shear flow: Part I, Comput. Math. Appl. 70 (2015) 345–352.
[37] K. Sawada, T. Kotera, A method for finding N-soliton solutions of the KdV equation and KdV-like equation, Prog. Theor. Phys. 51 (1974) 1355–1367.
[38] P.J. Caudrey, R.K. Dodd, J.D. Gibbon, A new hierarchy of Korteweg–de Vries equations, Proc. R. Soc. Lond. Ser. A 351 (1976) 407–422.
[39] D.J. Kaup, On the inverse scattering problem for cubic eigenvalue problems of the class $\psi_{xxx}+6Q\psi_x+6R\psi=\lambda\psi$, Stud. Appl. Math. 62 (1980) 189–216.
[40] B.A. Kupershmidt, A super Korteweg–de Vries equation: an integrable system, Phys. Lett. A 102 (1984) 213–215.

Please cite this article in press as: A.R. Seadawy, et al. Travelling wave solutions of the generalized nonlinear fifth-order KdV water wave equations and its stability, J. Taibah Univ. Sci. (2016), http://dx.doi.org/10.1016/j.jtusci.2016.06.002